



# Enhanced Computer Aided Simulation of Meshing and Contact with Application for Spiral Bevel Gear Drives

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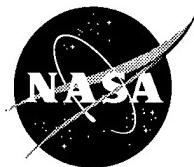
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# **Enhanced Computer Aided Simulation of Meshing and Contact with Application for Spiral Bevel Gear Drives**

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## **Summary**

An integrated tooth contact analysis (TCA) computer program for the simulation of meshing and contact of gear drives that calculates transmission errors and shift of bearing contact for misaligned gear drives has been developed. The computer program combines numerical solutions for the problems above and their graphical interpretation. The program is applicable for various gear drives but requires derivation of tooth surface equations for each gear drive type. The computer program represents a set of integrated operations such as the development of required algorithms, database storage, copying, deleting, printing, and design correction from the database. The computer program uses the Java programming language. An example for a spiral bevel gear drive is presented.

## **Nomenclature**

$\delta$	elastic deformation of the gear teeth
$(\Delta\phi_g^{(1)})$	predesigned parabolic function
$(\Delta\phi_g^{(2)})$	linear function of transmission errors
$(\Delta\phi_g^{(3)})$	resulting function of transmission errors as the sum $(\Delta\phi_g^{(1)} + \Delta\phi_g^{(2)})$
$\phi_i$	angle of rotation of pinion ( $i = p$ ) and gear ( $i = g$ )
$\phi_g(\phi_p)$	transmission function
$\psi_i$	generalized parameter of motion in the process of gear generation ( $j = t$ ) and pinion generation ( $j = c$ )
$\Sigma_p, \Sigma_g$	pinion and gear tooth surfaces
$C^m(m = 1, 2, \dots)$	designation of class function where all partial derivatives at least of order $m$ exist
$f = 0$	equation of meshing

$M_{ij}$	4x4 matrix for coordinate transformation from $S_c$ to $S_p$ ( $i = p, j = c$ ) and from $S_t$ to $S_g$ ( $i = g, j = t$ )
$N_1$	number of teeth of the pinion
$N_i$	normals to surfaces of pinion ( $i = p$ ) and gear ( $i = g$ )
$\mathbf{n}_f^{(i)}$	unit normals to surfaces of pinion ( $i = p$ ) and gear ( $i = g$ ) represented in coordinate system $S_j$
$P^0 = (u_c^0, \psi_c^0, \phi_p^0, u_t^0, \psi_t^0, \phi_g^0)$	initial set of parameters that satisfy equation (9)
$\mathbf{R}_i(u_j, \theta_j, \psi_j)$	vector function for family of generating surfaces represented in pinion coordinate system ( $i = p, j = c$ ) and gear coordinate system ( $i = g, j = t$ )
$\mathbf{r}_i(u_j, \theta_j)$	vector functions for representation in two parameter form of tooth surface of the pinion ( $i = p, j = c$ ) and the gear ( $i = g, j = t$ )
$\mathbf{r}_i(u_j, \theta_j)$	vector functions for pinion ( $j = c$ ) and gear ( $j = t$ ) generating surfaces
$S_c S_t$	Cartesian coordinate systems rigidly connected to pinion and gear tool surfaces
$S_p, S_g$ , and $S_f$	Cartesian coordinate systems rigidly connected to pinion, gear and housing
$(u_j, \theta_j)$	surface parameters of pinion ( $j = c$ ) and gear ( $j = t$ ) generating surfaces
$(u_j, \theta_j) \in E_i$	designation that indicates that parameters $(u_j, \theta_j)$ are located in area $E_i$
$v^{(ji)}$	relative (siding) velocity between the generating and the generated surfaces: (a) ( $j = t, i = g$ ) in case of gear generation; (b) ( $j = c, i = p$ ) in case of pinion generation

## Introduction

Computerized simulation of meshing and contact of tooth surfaces of a gear drive is currently a topic of research performed by many researchers (refs. 1 to 13). Such simulation is required for the following purposes: (i) testing the proposed design; (ii) for the prediction of the precision of manufacture and (iii) the assembly of the gear drive.

The simulation of meshing and contact of tooth surfaces requires the development of a complex computer program based on the following algorithms that represent:

- (1) Conditions of existence of an envelope to a family of surfaces generated by the tool surface.
- (2) Conditions of continuous tangency of pinion—gear tooth surfaces.
- (3) Determination of instantaneous contact ellipse of two surfaces being in point contact.
- (4) Formation of the bearing contact as a set of instantaneous contact ellipses.
- (5) Determination of transmission errors and the shift of the bearing contact caused by errors of alignment and manufacture.

Computer programs for simulation, called TCA (Tooth Contact Analysis), provide numerical solutions for the algorithms that have been developed. In addition to the database of numerical results the graphic interpretation of such results is required.

The local synthesis approach (refs. 5, 6 and 8) provides: (i) tangency of tooth surface at the mean contact point M, (ii) the direction of the tangent to the path of contact at M, (iii) the magnitude of the major axis of the instantaneous contact ellipse at M and (iv) the magnitude of the derivative of the parabolic function of transmission errors at M. Local synthesis permits the optimization of conditions for meshing and contact at M.

In this paper an integrated computer program for simulation is described for the combination of numerical solutions and their graphical interpretation. The application of such a computer program is illustrated with the example of a spiral bevel gear drive.

## Basic Algorithms

The algorithms used in the computer codes are contained in references 5, 6, 8 and 10. The basic concepts of the algorithms will be limited to a brief discussion in this paper, however more details are available in references 5, 6, 8, and 10.

### Determination of Envelope $\Sigma_i$ ( $i = p, g$ ) to Family of Tool Surfaces $\Sigma_j$ ( $j = c, t$ )

It is considered that the pinion and the gear tooth surfaces,  $\Sigma_p$  and  $\Sigma_g$ , are generated separately by application of generating surfaces  $\Sigma_c$  and  $\Sigma_t$ , that generate  $\Sigma_p$  and  $\Sigma_g$ , respectively. The tool surface is represented in parametric form by vector function  $\mathbf{r}_j(u_j, \theta_j)$  ( $j = c, t$ ). The family of tool surfaces is represented in coordinate system  $S_i$  ( $i = p, g$ ) (rigidly connected to the generated surface  $\Sigma_i$ ) by the following matrix equation

$$\mathbf{R}_i(u_j, \theta_j, \psi_j) = \mathbf{M}_{ij}(\psi_j) \mathbf{r}_j(u_j, \theta_j), \quad (i = p, g), \quad (j = c, t) \quad (1)$$

where  $\psi_j$  is the generalized parameter of motion.

The necessary condition of existence of an envelope  $\Sigma_i$  to family  $\mathbf{R}_i$  is determined as follows (refs. 6, 8 and 10)

$$\left( \frac{\partial \mathbf{R}_i}{\partial u_j} \times \frac{\partial \mathbf{R}_i}{\partial \theta_j} \right) \cdot \frac{\partial \mathbf{R}_i}{\partial \psi_j} = f(u_j, \theta_j, \psi_j) = 0 \quad (2)$$

The alternative approach for determination of  $f = 0$  is

$$\mathbf{N}_j \cdot \mathbf{v}_j^{(ji)} = f(u_j, \theta_j, \psi_j) = 0, \quad (i = p, g), \quad (j = c, t) \quad (3)$$

Here:  $\mathbf{N}_j$  is the normal to  $\Sigma_j$ ;  $\mathbf{v}_j^{(ji)}$  is the sliding velocity in the relative motion of  $\Sigma_j$  with respect to  $\Sigma_i$ . The envelope  $\Sigma_i$  is determined by equations (1) and (2) (or (1) and (3)) considered simultaneously. These equations represent surface  $\Sigma_i$  by three related parameters. Using equation of meshing (2) or (3), we may represent  $\Sigma_i$  in two parameter form as follows

$$\mathbf{R}_i(u_j, \theta_j(u_j), \psi_j) = \mathbf{r}_i(u_j, \psi_j) \quad (4)$$

Sufficient conditions of existence of envelope  $\Sigma_g$  require the existence of  $\Sigma_g$  as a *regular* surface. Determination of singularities of  $\Sigma_g$  is represented in references 4, 6 and 8.

### Conditions of Tangency of Pinion - Gear Tooth Surfaces

We consider three coordinate systems  $S_p$ ,  $S_g$ , and  $S_f$  rigidly connected to pinion  $p$ , gear  $g$ , and the housing of the drive, respectively. The tooth surfaces  $\Sigma_p$  and  $\Sigma_g$  are represented in coordinate systems  $S_p$  and  $S_g$ , respectively, by the following functions

$$\mathbf{r}_i(u_j, \psi_j) \in C^2, \quad \frac{\partial \mathbf{r}_i}{\partial u_j} \times \frac{\partial \mathbf{r}_i}{\partial \theta_j} \neq 0, \quad (u_j, \psi_j) \in E_i, \quad (i = p, g), \quad (j = c, t) \quad (5)$$

where  $\mathbf{r}_i(u_j, \psi_j)$  have derivatives to at least the second order, and  $(u_j, \psi_j)$  are contained in the vicinity of  $E_i$ .

The surface unit normals are represented as

$$\mathbf{n}_i = \frac{\frac{\partial \mathbf{r}_i}{\partial u_j} \times \frac{\partial \mathbf{r}_i}{\partial \psi_j}}{\left| \frac{\partial \mathbf{r}_i}{\partial u_j} \times \frac{\partial \mathbf{r}_i}{\partial \psi_j} \right|} \quad (6)$$

Using coordinate transformation from  $S_i$  to  $S_f$ , we represent the conditions of continuous tangency of surfaces  $\Sigma_p$  and  $\Sigma_g$  by the following equations

$$\mathbf{r}_f^{(p)}(u_c, \psi_c, \phi_p) = \mathbf{r}_f^{(g)}(u_t, \psi_t, \phi_g) \quad (7)$$

$$\mathbf{n}_f^{(p)}(u_c, \psi_c, \phi_p) = \mathbf{n}_f^{(g)}(u_t, \psi_t, \phi_g) \quad (8)$$

where  $\phi_p$  and  $\phi_g$  are the angles of pinion and gear rotation, respectively. Surfaces  $\Sigma_p$  and  $\Sigma_g$  are in point contact at every instant during meshing. Equations (7) and (8) yield only five independent scalar equations in six unknowns

$$f_k(u_c, \psi_c, \phi_p, u_t, \psi_t, \phi_g) = 0 \quad (k = 1, \dots, 5) \quad (9)$$

since  $|\mathbf{n}_f^{(p)}| = |\mathbf{n}_f^{(g)}| = 1$ .

Computer aided simulation of meshing is based on the following considerations:

(i) It is necessary to determine the first guess, a set of parameters

$$P^0 = (u_c^0, \psi_c^0, \phi_p^0, u_t^0, \psi_t^0, \phi_g^0) \quad (10)$$

that satisfy the system of nonlinear equations (9).

(ii) The goal is to determine the solutions of equation (9) in the neighborhood of  $P^0$  by functions

$$\{u_c(\phi_p), \psi_c(\phi_p), u_t(\phi_p), \psi_t(\phi_p), \phi_g(\phi_p)\} \in C^1 \quad (11)$$

where  $C^1$  means that each of the functions have derivatives at least to the first order.

Such functions exist in the neighborhood of  $P^0$ , if in accordance to the theorem of implicit function systems existence (ref. 2), the Jacobian of equation system (9) differs from zero, i.e. if

$$\frac{D(f_1, f_2, f_3, f_4, f_5)}{D(u_c, \psi_c, u_t, \psi_t, \phi_g)} \neq 0 \quad (12)$$

The solution of equations (9) by functions (11) is an iterative process. A Newton-Raphson method is used for the solution and is described in references 14 and 15.

### Determination of the Instantaneous Contact Ellipse

The orientation and dimensions of the contact ellipse requires: (i) the knowledge of elastic deformation  $\delta$  of gear teeth (assumed as known from experimental data), and (ii) the principal curvatures and directions at the instantaneous contact point. Then, as shown in reference 7, we may determine the magnitude of the axes of the contact ellipse and their orientation. A substantial simplification to the solution is provided if the principal curvatures and directions of the generated surface are expressed in terms of the principal curvatures and directions of the generating surface (the tool surface) (refs. 6 to 8). The bearing contact on the tooth surface is formed as the set of instantaneous contact ellipses.

## Reduction of Noise

Experiments have shown that the main source of vibration and noise are the errors of gear alignment (refs. 5, 6, and 8). The transmission function  $\phi_g(\phi_p)$  of a misaligned gear drive is a piecewise, almost linear function as shown in figure 1(a). A reduction of noise is obtained by transformation of the shape of the transmission function as shown in figure 1(b) that represents a sum of a linear function  $\phi_g^{(l)}(\phi_p)$  and a parabolic function for each cycle of meshing determined by  $\phi_p = \frac{2\pi}{N_p}$  where  $N_p$  is the number of teeth of the pinion,  $\phi_g^{(l)}(\phi_p) = \frac{N_p}{N_g} \phi_p$  (fig. 1(b)). The parabolic function of transmission errors (fig. 2) is determined as

$$\Delta\phi_g(\phi_p) = -a\phi_p^2 \quad (13)$$

where  $a$  is the parabola coefficient. Equation (13) must be predesigned for spiral bevel gears by mismatch of generating surfaces and by modified roll of the pinion or the gear that provides a nonlinear relation between the angles of rotation of the cradle of cutting machine and the angle of rotation of pinion (or gear) being generated. Shown in figure 2 is an example of the transmission error curves represented on an exaggerated scale. It is important to note that a predesigned parabolic function of transmission errors absorbs linear functions of transmission errors caused by misalignment. (Proven in (ref. 8)). This is illustrated in figure 3 that show that the sum of functions

$$\Delta\phi_g^{(1)} = -a\phi_p^2, \quad \Delta\phi_g^{(2)} = b\phi_p \quad (14)$$

indeed represents a parabolic function

$$\Delta\phi_g^{(3)} = -a(\phi_p^*)^2 \quad (15)$$

with the same parabola coefficient  $a$ . Function (15) is the *resulting* parabolic function of transmission errors after the absorption of the undesired linear function of transmission errors. The origin of coordinate axes  $(\Delta\phi_g^{(3)}, \phi_p^*)$  is shifted by  $(c, d)$  with respect to the origin of coordinate axes  $(\Delta\phi_g^{(1)}, \phi_p)$ .

## Localization of Bearing Contact

The main ideas of the localization of the bearing contact are based on the following considerations (refs. 6, 8 and 10):

- (i) The instantaneous contact of tooth surfaces along a line is substituted by the instantaneous point contact of surfaces.
- (ii) Point contact of surfaces is provided by crowning of surfaces in two directions: the profile direction (across the surface), and the longitudinal direction.
- (iii) The crowning of surfaces is obtained by the proper mismatch of generating surfaces.
- (iv) In addition to localization of the bearing contact, a predesigned parabolic function of transmission errors is provided for the absorption of linear functions of transmission errors caused by errors of alignment.

The ideas of localization of bearing contact described above are illustrated with an example for a spiral bevel gear drive.

# Enhanced Computer Program for Simulation of Meshing and Contact

Most of the existing computer programs for simulation of meshing and contact require the following steps:

- (i) A numerical solution of the needed algorithms requires developing a computer program. Also, this permits use the mathematical libraries developed for the computer language chosen. After execution of the computer program, the numerical results are stored in text form in a sequence of files.
- (ii) The files with numerical results are manipulated with a graphics software package to interpret the output of the computer program. The output is given by graphs of the transmission errors and tooth bearing contact for an aligned and misaligned gear drive.
- (iii) An investigation of the influence of errors of alignment requires additional computation and manual or semi-automatic modification of the output files.

The execution time for the computer program on a modern PC-class computer is very short but the graphic interpretation of the obtained numerical results by the graphics software requires manual actions by the researcher.

A computer program that combines the numerical computations and graphical interpretation of numerical results in one computer program was developed. The computer program provides a set of integrated operations such as the development of the algorithms, storing in a database, copying, deleting, printing, design correction from the database, etc. The computer program that has been developed is based on application of the Java programming language for the following reasons:

- (1) It permits Graphical User Interface (GUI) program development which are platform independent: they can be run on various platforms (PC, Unix, Main Frame Machines, etc.) without modifications.
- (2) It permits programming with a DataBase Management System (DBMS) in a consistent and easy way. The management system gives us the opportunity to implement operations for storing, correction of design, and obtaining the graphical output for several stages of meshing simulation.
- (3) It provides excellent support for distributed computing, and it is useful for implementation of our system based on client-server architecture (*client* is responsible for graphical results interpretation, *server* does the computation). By separating the system into two parts, a better utilization of computing resources is achieved. The server may be run on a fast computer and carry out extensive computation while the client may be run on low cost machines (thus this work distribution provides a better utilization of the computing resources), and the modification of the system is simplified (each part can be modified independently).

## Description of the System Developed

Figure 4 represents the architecture of the system, the set of components that form the system and their interaction. These components are: Computation Unit, Plug-in Manager, Graphical User Interface Subsystem, adaptors for various databases (Gear Types Database, Designers Database, Design Database) and the Databases mentioned above.

**Computation Unit.**—This unit is the computer code that is designated for the numerical computation required for simulation of meshing and contact of one type of gear drive. A set of various Computation Units is assigned for various types of gear drives. By establishing certain formats for input and output of data for these computer codes, we can add to the system new software for gear drives without modification of the existing Java framework.

**Plug-in Manager.**—This unit is an intermediate link in the Logic Processing that is responsible for the invocation and termination of the program (Computation Unit) for further operations. The separation of the computer code from the Java graphical framework (Graphical User Interface) makes the system more adaptive to the changes of the computer code. Modification of these programs will not cause the so-called ripple effect, a cascade of undesirable changes in other components of the system.

The other advantage of the application of the Plug-in Manager component is the ability to decouple the computation server and the Java graphical part for the following purposes:

- (i) To run the computer code on a fast-server class computer.
- (ii) In case of application of a large enough number of computer programs (for various gear drives), it is useful to assign a computation server where all these programs will be stored and executed. A multitude of low

cost computers with the components of Plug-in Manager, Graphical User and other components of the system (fig. 4) can be applied.

**Graphical User Interface Subsystem.**—This unit is designated for the interaction with the Computation Unit (computer code) that implements the numerical solutions of the developed algorithms. The goal of interaction is the graphical interpretation of the numerical solutions. The graphs represent the bearing contact and the transmission errors of aligned and misaligned gear drives. The client has a set of four windows for graphical output (graphical result viewer) (fig. 5).

The transmission errors are represented in the window to an assigned scale (fig. 5). Functions of transmission errors are determined for three meshing cycles. The beginning and the end of the cycle of meshing are the points of intersection of the neighboring functions of transmission errors. The maximum value of transmission errors is determined as the ordinate of the function at the beginning (or end) of cycle of meshing. The program user also can zoom in on the maximum value of transmission errors by magnification of function of errors at the transfer point of the function intersection. Also, the transmission errors can be calculated and displayed for various types of misalignment.

Two windows are designed for visualization of the path of contact and the formation of the bearing contact as the set of segments, each of which represents the major axis of the instantaneous contact ellipse. The shift of bearing contact caused by misalignment can also be displayed.

**Database and Database Adapters.**—The components of the software of this unit are designated for the following purposes:

- (i) To store information about various types of gear drives (Gear Type Database).
- (ii) To store the information related to various designers, including attributes such as designers name, password, and various preferences (Designers Database).
- (iii) To store the information about the different designs of different gear types (input data, output files, access rights, etc.) (Design Database).

Database Adapters separate the GUI subsystem from this part of the Database Management System that is designated for the implementation of Databases. Such decoupling is beneficial for the adaption of the system to possible modifications.

The designed system permits various operations related to the design of the gear drive and the simulation of meshing. The operations are as follows:

- (i) *Create New Design* running the computer code for a new set of design parameters.
- (ii) *Save Design* storing the data in the Design Database (including the input data).
- (iii) *Open Design* operation that permits access to the stored design data, then modify the input data, recalculate the characteristics of the gear drive with modified design data and, if it is necessary, store it again.
- (iv) *Delete Design* operation.

*Print Design* permits printing the design input and the graphical output

- (vi) *Give Input Data* operation that allows the designer to get access to the input data and some algorithm-related parameters (for instance the first guess) and then change the data (fig. 6).
- (vii) *Start Computation* command for starting the computation process.

## Example: Application for Design and TCA of Face-Milled Spiral Bevel Gears

The input data for the example design is given in figure 6. The gear machine-tool settings are adapted from the Gleason summary (ref. 16). The pinion machine-tool settings are optimized to obtain: (i) a low level of transmission errors, (ii) a parabolic type of function of transmission errors, (iii) a chosen direction of path of contact, and (iv) designated length of major axis of contact ellipse. The optimization of pinion machine-tool settings is achieved by application of local synthesis approach as described in reference 5. The crowning of the pinion tooth surface is achieved by the mismatch of pinion generating surface.

This approach allows various directions of the path of contact and the bearing contact, respectively. Two preferable directions of the path of contact, the profile direction and longitudinal direction are considered.

The results of computation for the path of contact in profile direction are shown graphically in figures 7 and 8 that illustrate the transmission errors and bearing contact of an aligned gear drive, at the toe, mean, and heel contact, respectively. The results obtained confirm that a low level of transmission errors is achieved

(approximately, not more than 8 arc sec) a parabolic type of transmission errors is provided, and the bearing contact is stable.

The results of computation for the case of a longitudinal direction of the path of contact are represented in figures 9 to 11. The results obtained are similar to the results represented in figures 7 and 8 for the profile direction of the path of contact.

## Summary

Based on the results contained in this report the following statements can be made:

- (1) An advanced design and simulation tool for the meshing and contact of spiral bevel gears has been described (conceptually).
- (2) An enhanced integrated computer program for tooth contact analysis (TCA) was described.
- (3) An example using the technique that has been developed is presented for spiral bevel gears. Results indicated a stable bearing contact and low parabolic transmission errors for spiral bevel gears for profile and longitudinal paths of contact.

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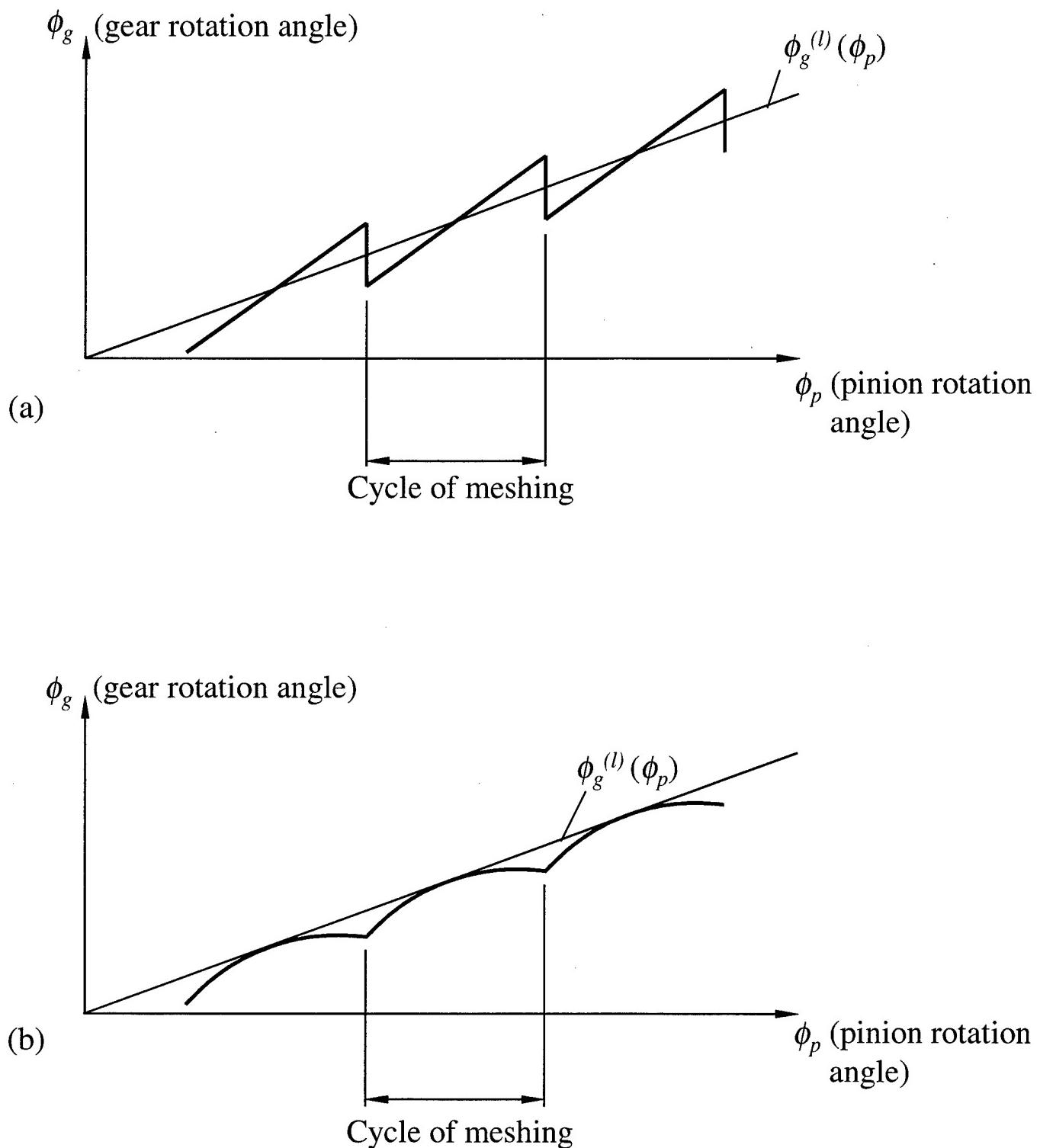


Figure 1. Transmission functions: a - piecewise function with linear function of transmission errors; b - piecewise function with parabolic function of transmission errors

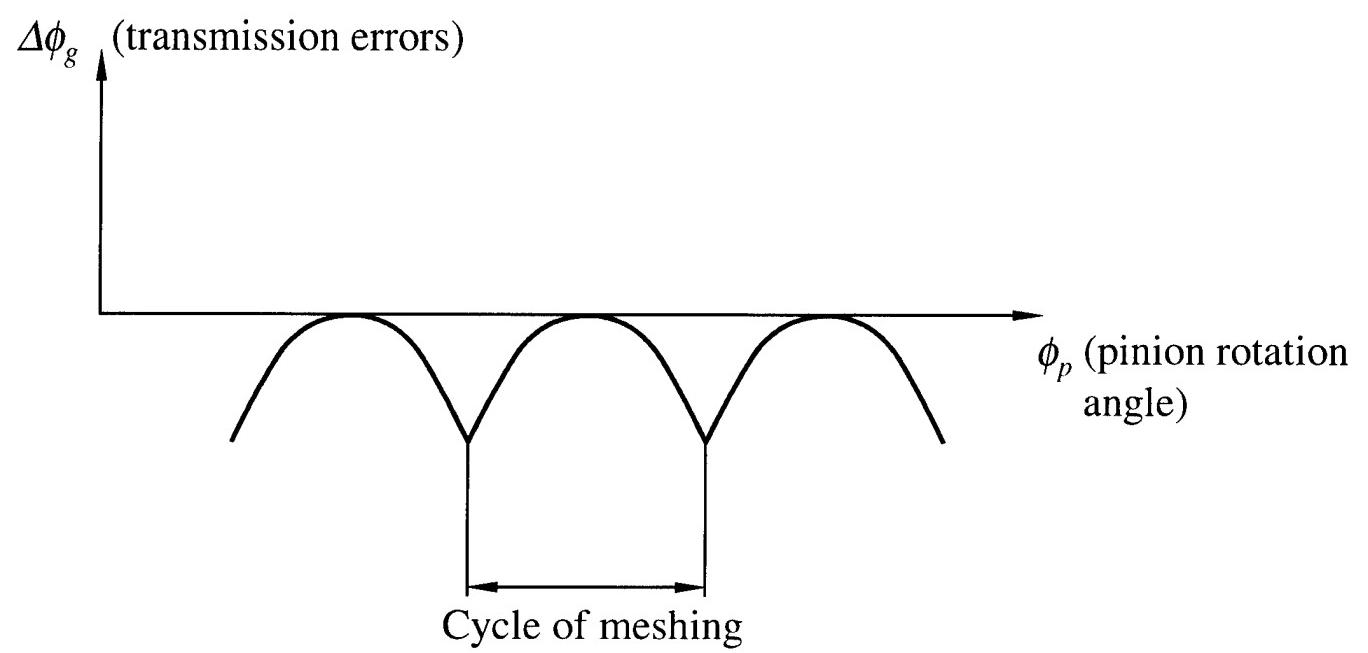


Figure 2. Parabolic function of transmission errors

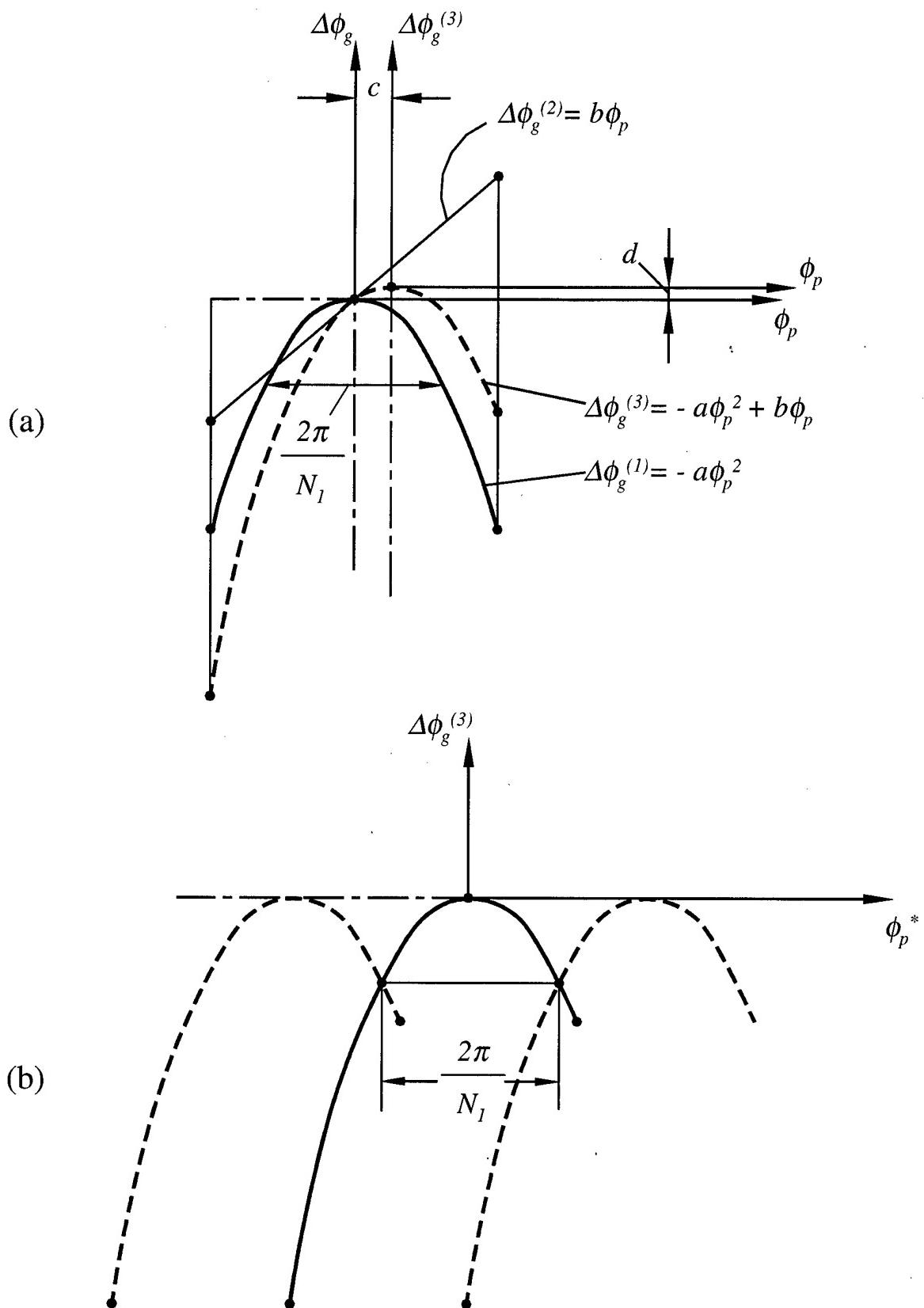


Figure 3. Interaction of parabolic and linear functions of transmission errors: (a) predesigned parabolic function and linear function of transmission errors; (b) resulting parabolic function of transmission errors

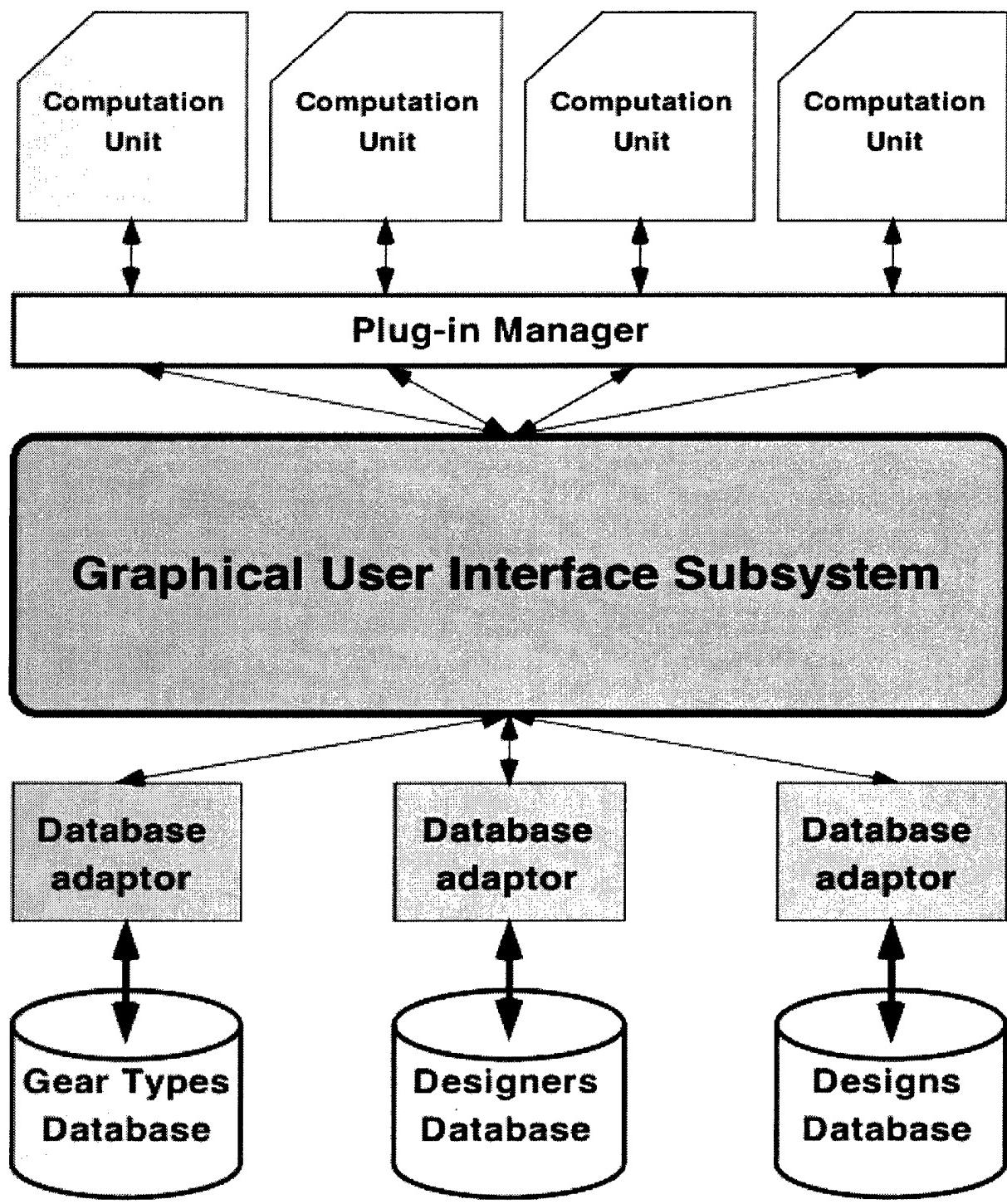


Figure 4. System architecture

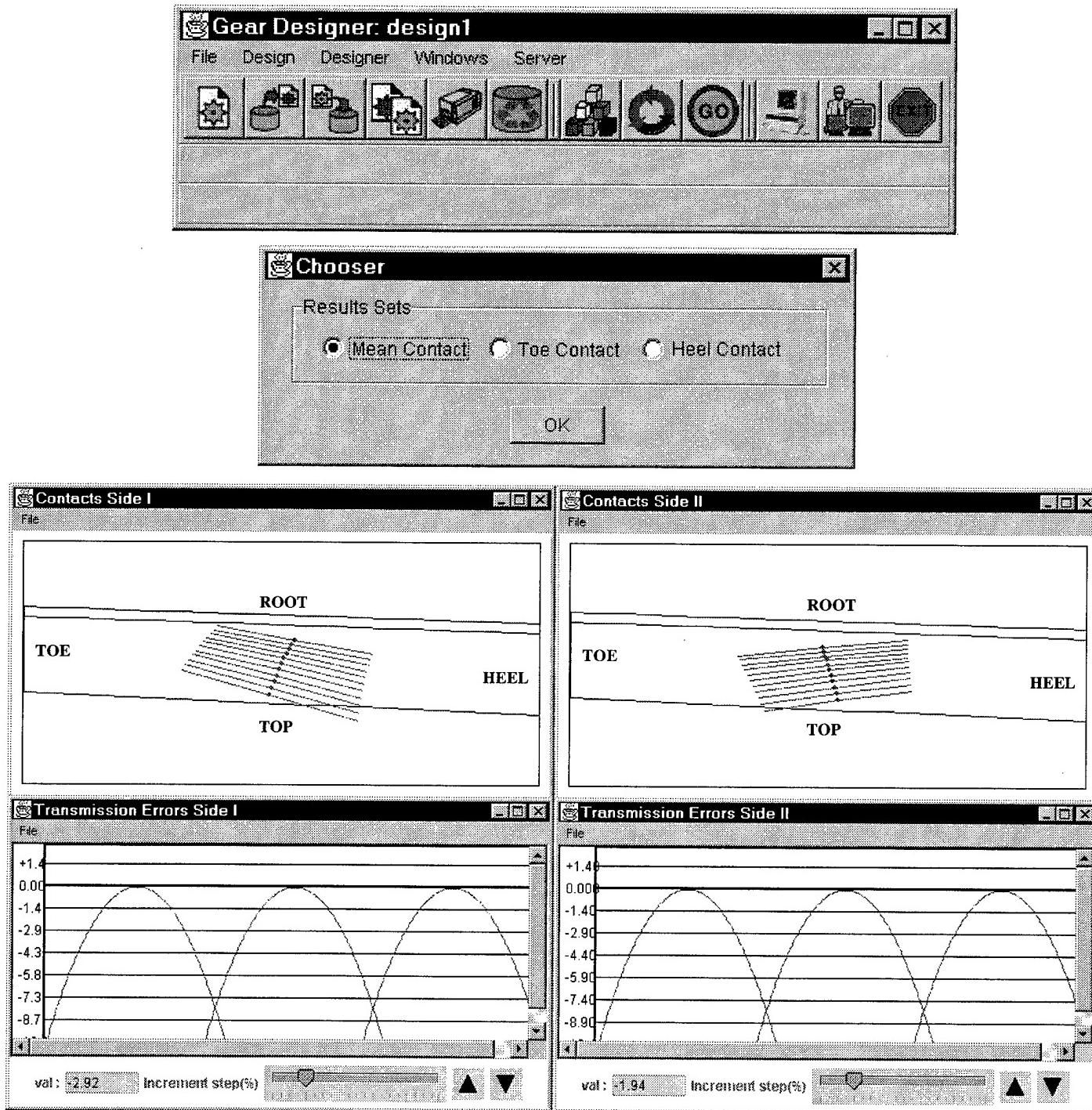


Figure 5. Profile direction of path of contact: Transmission errors and bearing contact for aligned gear drive

**Input Parameters**

General Parameters | Gear Machine Settings | Cutter Specifications | Design Parameters

Hand of Gear Spiral     Right Hand     Left Hand

Number of Pinion Teeth	27.0	Number of Gear Teeth	79.0	Shaft Angle (degrees)	51.18333
Whole Depth (mm)	7.1374	Gear Dedendum (mm)	5.4356		
Pinion Clearance (mm)	0.762	Face Width (mm)	39.878		
Mean Cone Distance (mm)	232.537	Mean Spiral Angle (degrees)	25.0		
Pinion Root Cone Angle (degrees)	14.06667	Pinion Face Cone Angle (degrees)	14.66667		
Gear Root Cone Angle (degrees)	36.51667	Gear Face Cone Angle (degrees)	37.11667		

Figure 6. Table of input data for design of face-milled spiral bevel gears

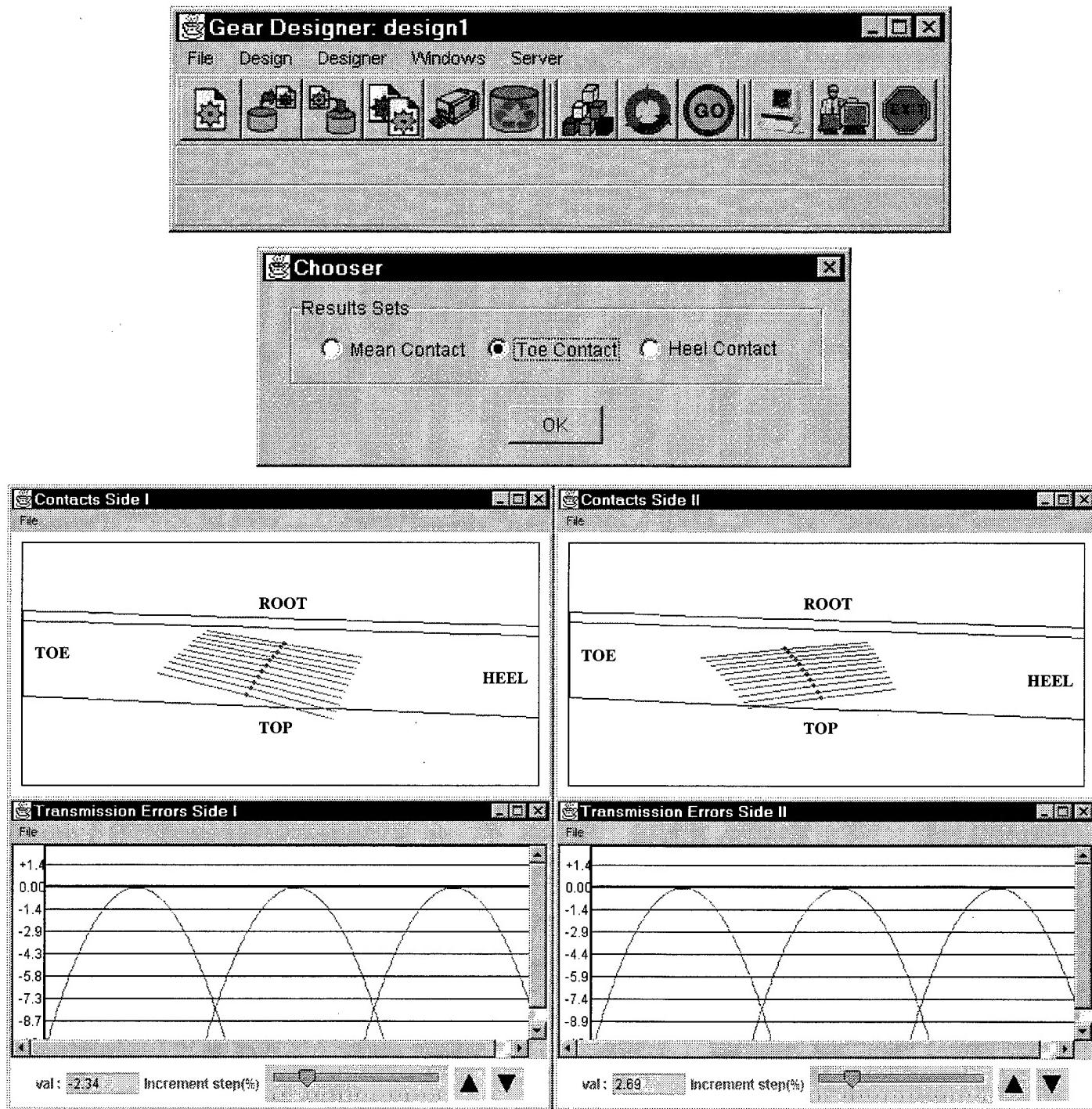


Figure 7. Profile direction of path of contact: Transmission errors and bearing contact in case of toe contact

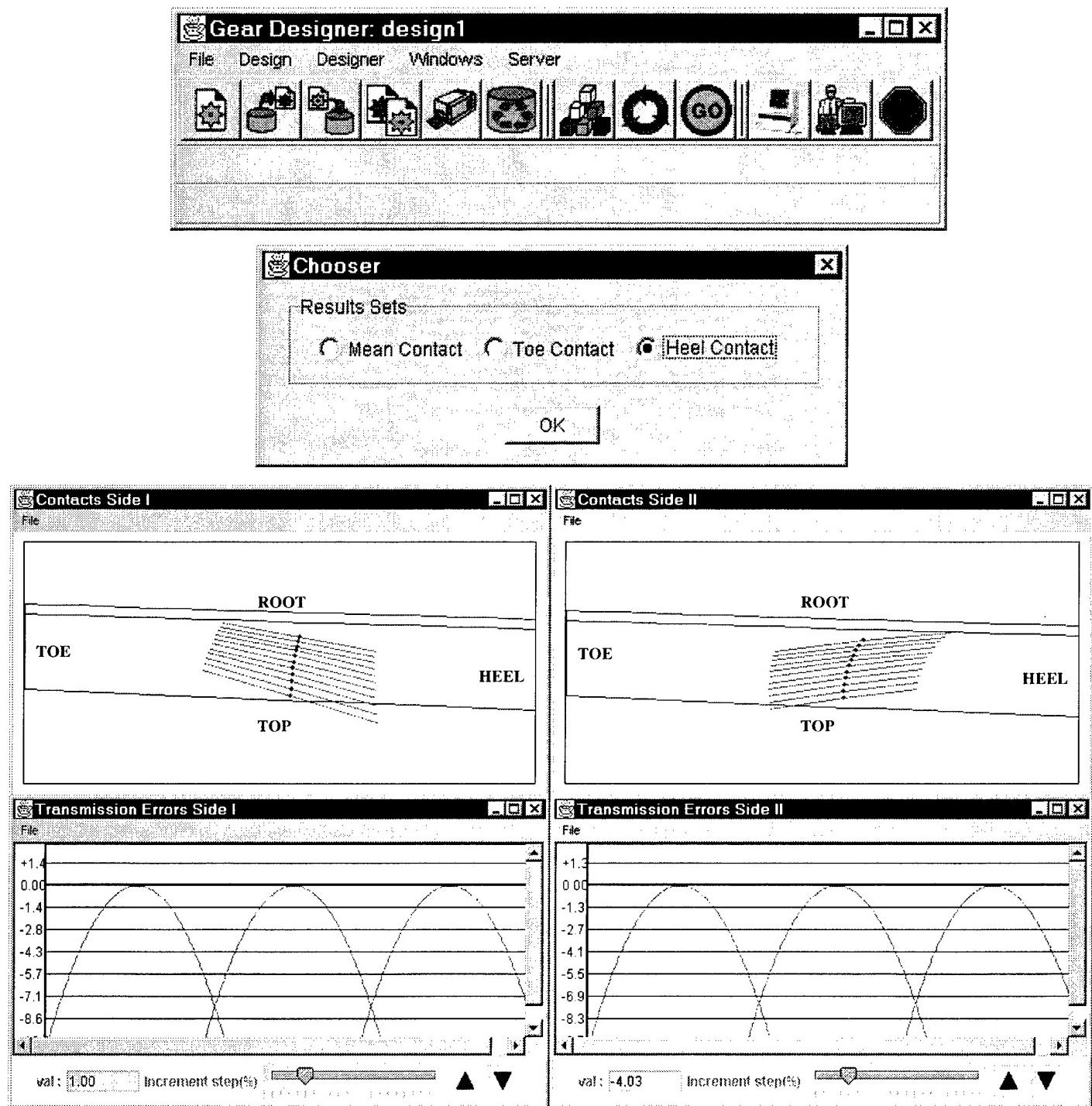


Figure 8. Profile direction of path of contact: Transmission errors and bearing contact in case of heel contact

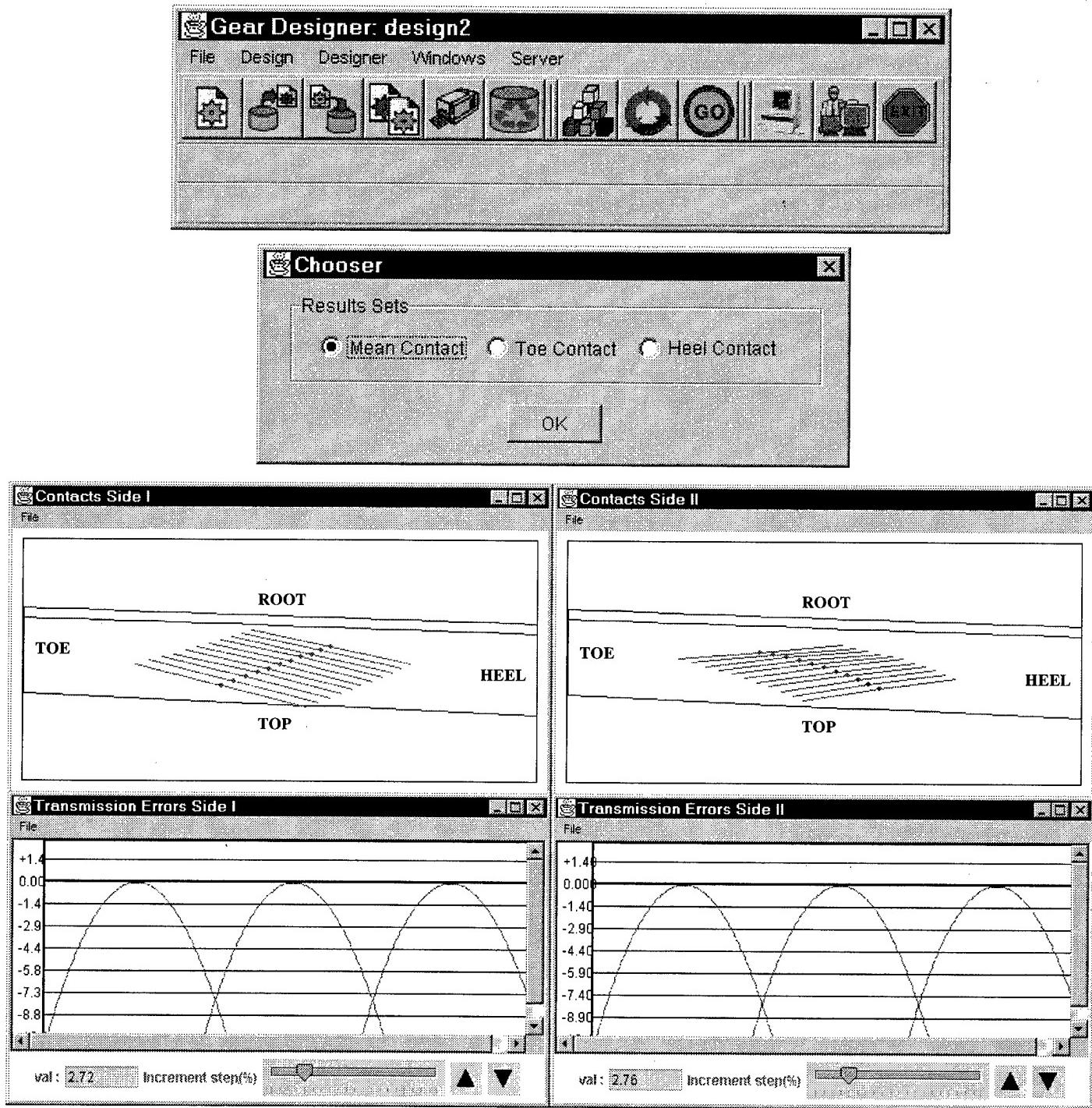


Figure 9. Longitudinal direction of path of contact: Transmission errors and bearing contact for aligned gear drive

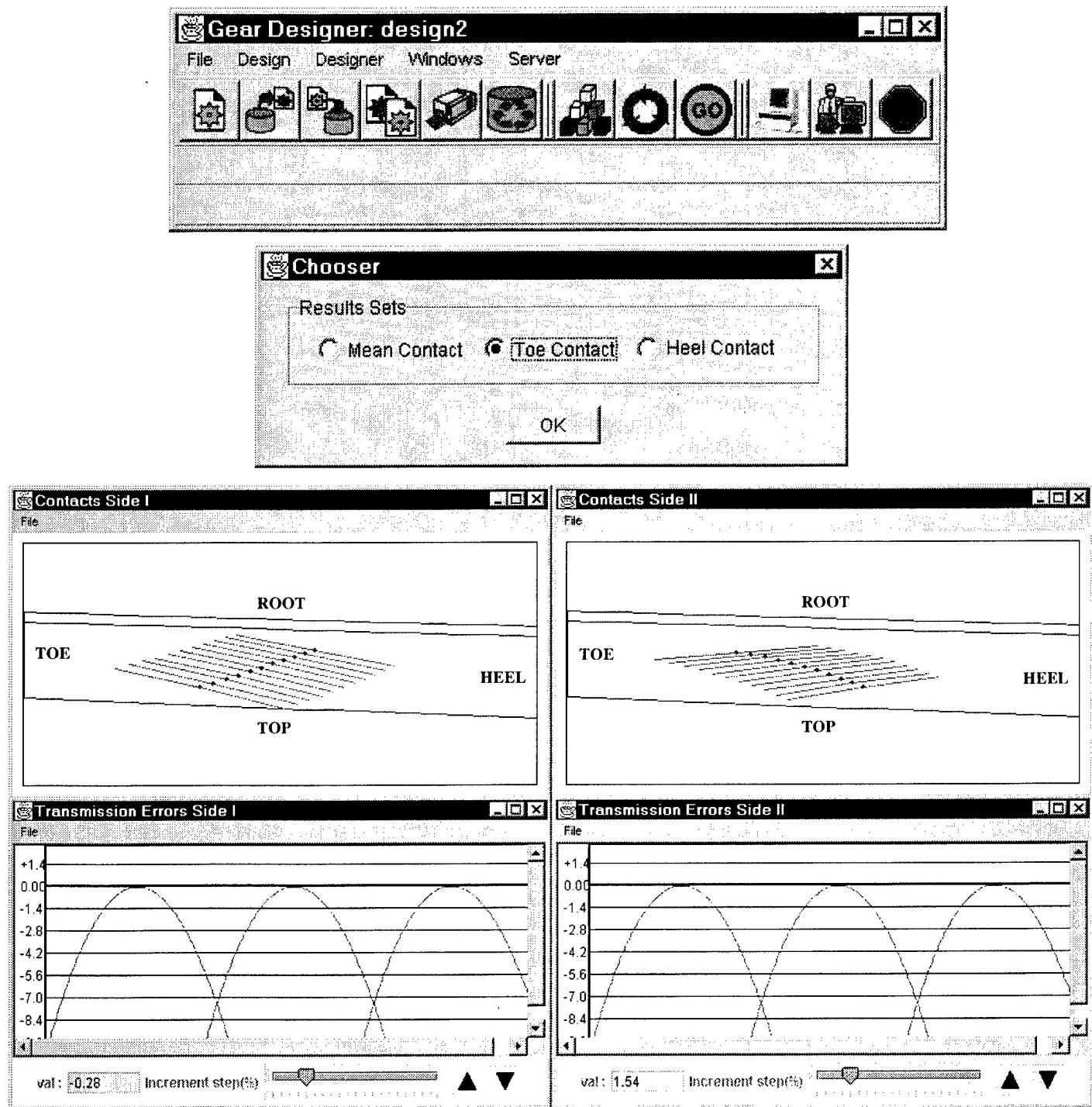


Figure 10. Longitudinal direction of path of contact: Transmission errors and bearing contact in case of toe contact

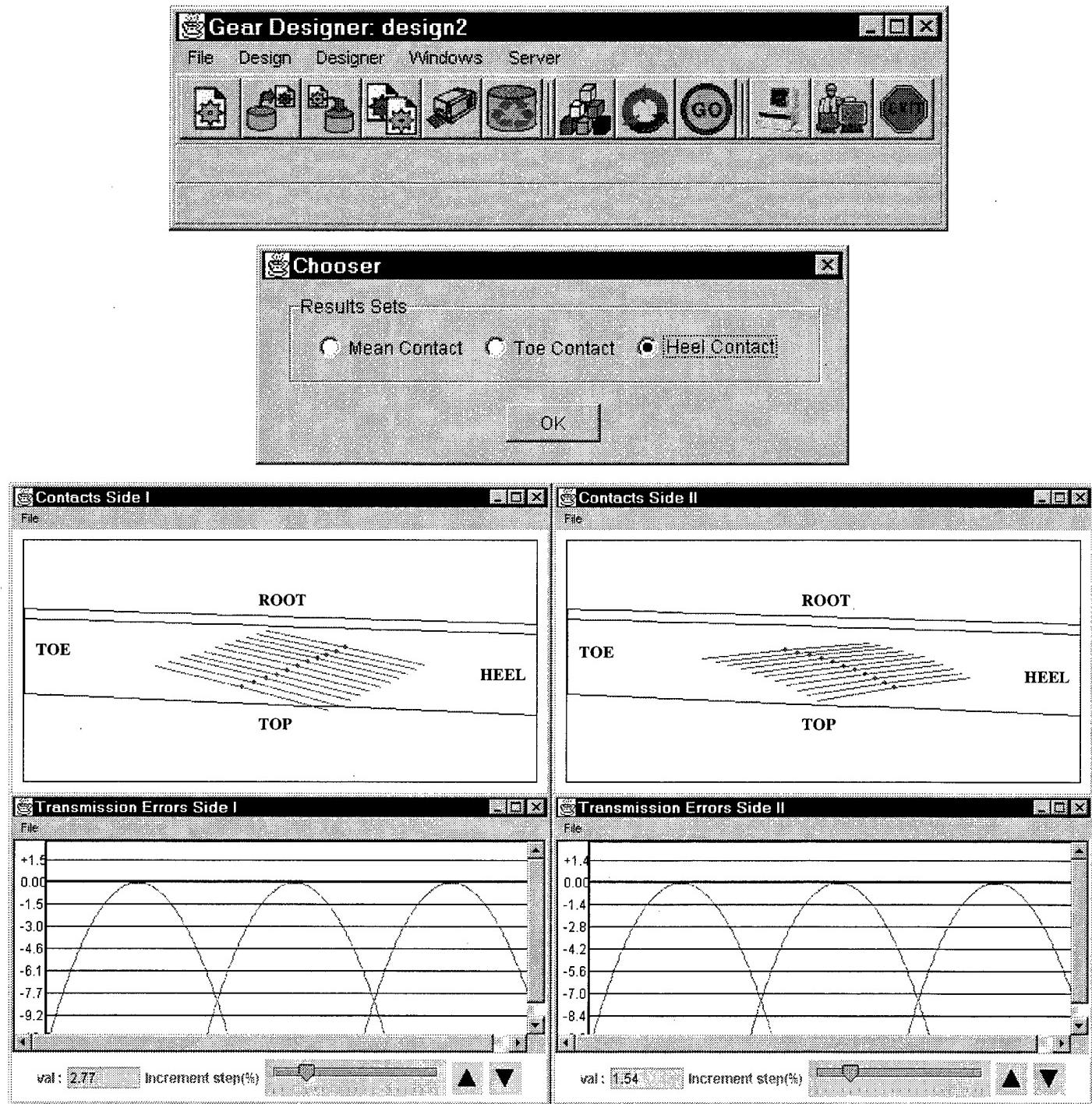


Figure 11. Longitudinal direction of path of contact: Transmission errors and bearing contact in case of heel contact

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<b>13. ABSTRACT (Maximum 200 words)</b>  An integrated tooth contact analysis (TCA) computer program for the simulation of meshing and contact of gear drives that calculates transmission errors and shift of bearing contact for misaligned gear drives has been developed. The computer program combines numerical solutions for the problems above and their graphical interpretation. The program is applicable for various gear drives but requires derivation of tooth surface equations for each gear drive type. The computer program represents a set of integrated operations such as the development of required algorithms, database storage, copying, deleting, printing, and design correction from the database. The computer program uses the Java programming language. An example for a spiral bevel gear drive is presented.			
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